Determination of Coherence Length and Coherence time for a Diode Laser Using a Czerny-Turner Monochrometer

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Abstract

All lasers have a coherence time and length as a result of the dimensions of the optical cavity in which the beam is formed. These values can be determined using a monochrometer to find the wavelengths present in the laser light emitted and the wavelength and frequency between modes. Using a Czerny-Turner monochrometer, the wavelength of a red diode laser was found to be 660 $\pm$ 6 nm with a coherence length of $1.08\times10^{-4} \pm 10.59 \times10^{-4}$ m and a coherence time of $3.6\times10^{-13} \pm 35.3\times10^{-13}$ s.

I. INTRODUCTION

Lasers produce light by a process called stimulated emission. The word "laser" is an acronym for "Light Amplification by Stimulated Emission of Radiation."[4] Lasers are capable of converting incoherent light into a coherent beam. In a normal chamber-produced laser, such as a HeNe laser, the gas in the optical cavity is heated to cause the atoms of the gas to start absorbing and releasing photons of specific wavelengths. Thus the possible wavelengths of a laser are determined by the temperature of the gas and the atoms that constitute the gas in the optical cavity. However, in the contained space of the optical cavity a phenomenon known as population inversion results, in which there are more electrons in the excited state than in the ground state, disrupting the equilibrium of the gas atoms. As a result, when one electron releases a photon and moves back down to the ground state, it is likely to cause produce the same result in another atom. This effect is the stimulated emission referred to in the "laser" acronym.[5]

With this process occurring, the optical cavity length and mirrors at either end are tuned to allow the chamber to function as a Fabry-Perot resonator. The mirrors by the use of specific materials to reflect only at a specific wavelength and the optical cavity length is a large integer multiple of that wavelength.[4] However, as might be guessed, there are other wavelengths that can resonate constructively at integer multiples within the given cavity length, and the mirrors will not completely cut out all reflection of these wavelengths. This is where the use of an monochrometer comes in. The monochrometer, in the case of this experiment a crossed Czerny-Turner monochrometer, separates the laser into its constituent modes corresponding to each wavelength present and allows for the measurement of the wavelength separation within each mode by observation of the fringe pattern. These observations then allow for the calculation of the coherence time and coherence length of the laser beam, which describe the amount of time and length, respectively, over which the light constituting the laser beam remains coherent. In this experiment, a diode laser is used. While diode lasers do not use a gas to produce the effect of stimulated emission, the same tuning characteristics are at play and will give the diode laser a coherence length and time analogous to those produced by a HeNe laser.

II. THEORY

All lasers display a characteristic coherence time (τ_{Coh}) and length (l_{Coh}). These characteristics are the result of the process of stimulated emission and optical cavity variables as discussed above. To determine τ_{Coh} and l_{Coh}, a monochrometer can be used. Monochrometers separate out light of a single wavelength into spectral lines and will separate light of multiple wavelengths into longitudinal modes composed of only one wavelength. Monochrometers accomplish this by using both spherical mirrors and a reflection grating along with an entrance slit and imaging surface as seen in Fig. (2). The first spherical mirror is aligned such that the laser light coming through the entrance slit is at the mirror's focal point given by Eq. (1) where

$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o} \tag{1}$$

in which f is the mirror's focal length, s_i is the image distance, and s_o is the object distance. The result is that the rays of light emerging from the first mirror are parallel and thus will be parallel when incident on the reflection grating, allowing it to separate each ray into its component modes and, due to the size of the slits in the grating, allowing for the wave nature of light to take effect and for the patterns of constructive and destructive interference that form the spectrum for each wavelength. Using basic trigonometric rules, the incident angle (θ_i) and reflected angle (θ_m) at the reflection grating can be calculated for each mode. θ_m and θ_i are then related to the wavelength, λ, of a given mode, m, by Eq. (2).

$$m\lambda = a(\sin\theta_m - \sin\theta_i) \tag{2}$$

where a is the spacing between lines in the diffraction grating. For a specific mode, the calculated wavelength can then be used along with the wavelength spacing, $\Delta\lambda$, and the index of refection of the semiconductor, n_{semi}, in the diode laser to determine the optical cavity length and the frequency spread, as seen in Eq. (3) and (4) where

$$\Delta\lambda = \frac{n_{semi}\lambda^2}{2L} \tag{3}$$

and

$$|\Delta\nu| = \frac{c}{n_{semi}\lambda^2}|\Delta\lambda| \tag{4}$$

where c is the literature value of the speed of light from NIST.[2] $\Delta\nu$ is then the inverse of τ_{Coh} and the coherence length can be determined by Eq. (5) where

$$l_{Coh} = c\tau_{Coh} \tag{5}$$

Along with the coherence time and coherence length, the length of the optical cavity, L, in the diode laser can also be calculated using Eq. (6) where

$$L = \frac{n_{semi}\lambda^2}{2\Delta\lambda} \tag{6}$$

where n_{semi} is the index of refraction of the semiconductor used in the diode laser.

III. METHODS AND MATERIALS

For this experiment, the first step was to measure the focal lengths of the spherical mirrors used in the monochrometer. Because setting an object just beyond the factory specification for the focal length of 101.6 cm would produce an image distance too large to practically measure and compare to the predicted image distance in the laboratory, a more practical method was used of placing an image source and mirror at opposite ends of a two meter long measuring stick and a plane surface was adjusted until the image of the source was in focus on the surface. The distance from the mirror to the focused image plane was 204.80 $\pm$ 0.05 cm. Using Equation (1) and the source and image distances, the calculated focal length of the mirror was 101.19 $\pm$ 0.04 cm.

Once the focal lengths of the mirrors were determined, the monochrometer was set up according to Figure (2). The diode laser was mounted in a stand directly behind the Entrance Slit. The distance between the Entrance Slit and Mirror 1, as well as the distance between Mirror 2 and the Array Detector, were set equal to the focal length of the mirrors at 101.19 $\pm$ 0.04 cm. The grating was then set in a position to place the zeroth order of diffraction on the imaging plane when the grating was at zero degrees and still remain out of the way of the laser's path. This position was at 93.90 $\pm$ 0.05 cm away from Mirror 2 and 94.20 $\pm$ 0.05 cm away from Mirror 1, with Mirror 1 and Mirror 2 being 60.80 $\pm$ 0.05 cm from each other. Throughout this process the height of the mirrors, grating, and entrance slit were adjusted to keep the path of the laser beam parallel to the optical table throughout the entire path of the optical system. With this setup in place, the diffraction grating was rotated to display the m = 1 or m = –1 longitudinal modes of the diode laser. Because the false signal for these modes falls on the line normal to the diffraction grating, it was possible to place another imaging plane at the false signal and measure the distance between the diffraction grating and the false signal to act as the base of the right triangle in the calculation of θ_m and θ_i, with the distance between the grating and the respective mirror forming the hypotenuse of the triangle, as seen in Fig. (??). From these values, the wavelength of the mode can be determined using Eq. (2).

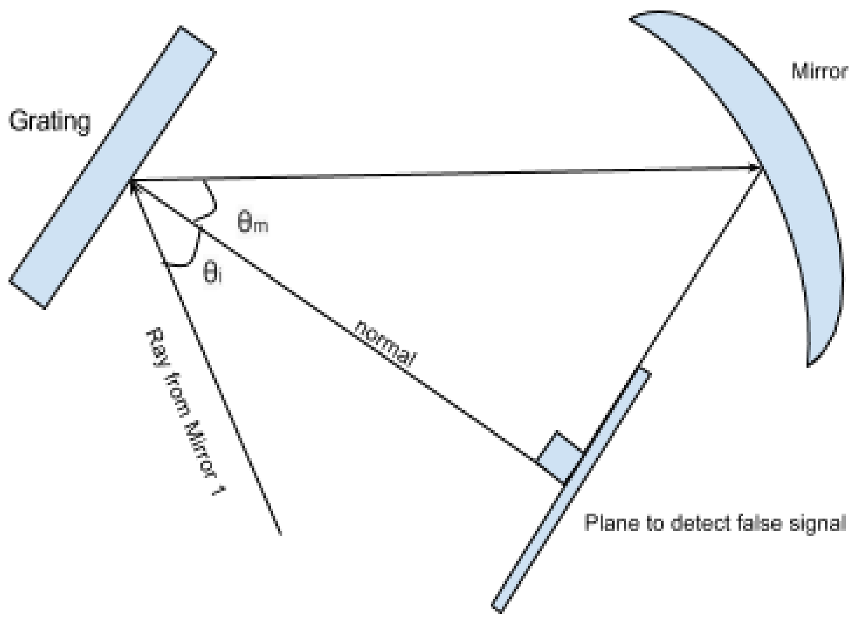

Figure 1. This figure shows the schematic of the method used to measure the distance of the normal used in the inverse cosine calculation of θ_m.

After the value of λ was determined, the grating was rotated to display the m = 2 mode at the image plane in order to take advantage of the larger and clearer spacing between fringes to aid in measurement and calculation of the wavelength spread of the laser. In order to calculate $\Delta\lambda$, the spacing between fringes, $\Delta\lambda$ was measured by performing multiple trials of tracing a specific length of the interference pattern and then counting the number of fringes recorded within that length. The total length divided by the number of lines gives the value of $\Delta\lambda$. From this value, along with the wavelength of the laser and the distances measured in the setup, τ_{Coh} and l_{coh} can eventually be arrived at, as described below.

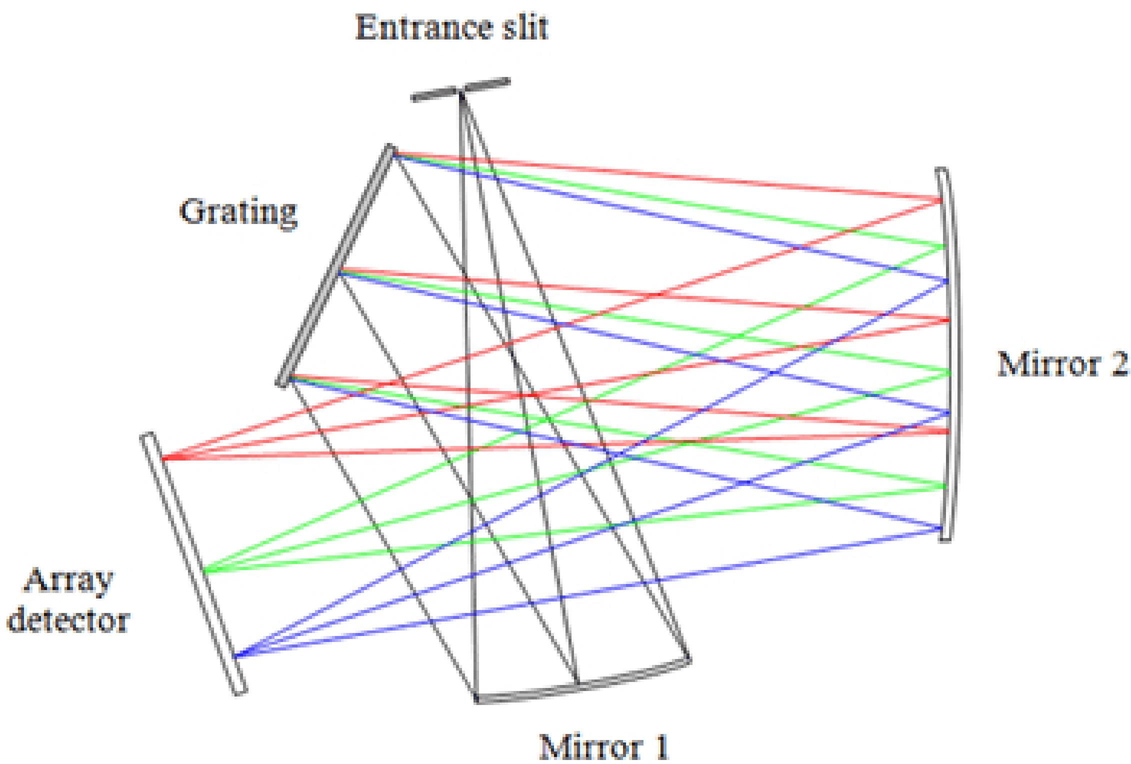

Figure 2. This figure shows the schematic of the crossed Czerny-Turner monochrometer used in the experimental design for the measurement of λ and $\Delta\lambda$. The distance between the Entrance slit and Mirror 1, as well as the distance between Mirror 2 and the Array Detector (the image plane) is equivalent to the focal length of the spherical mirror. Image courtesy of B+W Tech.

IV. DATA PRESENTATION AND ANALYSIS

To determine the value of λ for the laser, the grating was first rotated to display the m = 1 mode. Forming the right triangle described above, the distance from the grating to the false signal at the normal was 93.70 $\pm$ 0.05 cm. Using the inverse the distance from the grating to Mirror 1 as the hypotenuse for finding θ_i, the inverse cosine resulted in a calculated value of 5.906 $\pm$ 0.007 degrees for θ_i. In order to arrive at θ_m, the angle at the grating forming the peak of the triangle between Mirrors 1 and 2 and the grating must be added to θ_i. This angle is calculated by doubling the angle formed by splitting this triangle into two right triangle, one with Mirror 1 at the corner of the opposite side and hypotenuse and the other with Mirror two at the corner of the opposite side and the hypotenuse. Using the distance to the halfway point between Mirror 1 and Mirror 2, the distance from the grating to this point, and the inverse tangent, the angle at the peak of the triangle is calculated to be θ_{tot} = 37.655 ± 0.003 degrees. θ_m then is equal to θ_i + θ_{tot}, such that θ_m = 43.561 ± 0.008 degrees. Because θ_i is on the same side of the normal as θ_m, θ_i is assigned a negative value in Eq. (2). This results in a calculated value of λ = 660 ± 6 nm. This process is repeated again for the m = –1 mode, resulting in values of θ_i = 42.236 ± 0.009 degrees, θ_m = 4.581 ± 0.009 degrees, and λ = 627 ± 0.007 nm. The values of the distances, angles, and resultant λ values can be seen below in Table (I).

Table I. This table shows the distances used to calculate the angles of incidence and reflection for each mode along with their resultant wavelengths or, in the case of mode m = 2, the resultant wavelength spread.

m	L_{normal} (cm)	L_{mirror} (cm)	θ_i (deg)	θ_m (deg)	λ (nm)
+1	93.70 ± 0.05	94.20 ± 0.05	5.906 ± 0.007	43.561 ± 0.008	660 ± 6
−1	93.60 ± 0.05	93.90 ± 0.05	42.236 ± 0.009	4.581 ± 0.009	627 ± 7
m	L_1 (cm)	L_2 (cm)	θ_{m1} (deg)	θ_{m2} (deg)	$\Delta\lambda$ (nm)
2	79.70 ± 0.05	101.20 ± 0.05	69.868 ± 0.003	69.974 ± 0.003	0.186 ± 1.053

Moving to the m = 2 mode, the average of multiple trials of the measurement of Δx, which can be found in Table (II), results in a value of 0.131 ± 0.005 cm for the spacing between fringes. In order to find the spacing as an angle Δx_{avg} can be approximated as the numerator of $\tan \Delta\theta$ where the denominator of the function is the distance between Mirror 2 and the imaging plane. These distances and angles can also be seen in Table (I). $\tan \Delta\theta$ can then be approximated to $\Delta\theta$, resulting in a calculated value of $\Delta\theta$ = 0.74 ± 0.003 degrees. Using this value, Eq. (2) can be modified to

$$\Delta\lambda = \frac{a(\sin\theta_{m1} - \sin\theta_{m2})}{m} \tag{7}$$

where θ_{m2} is the equal to θ_{m1} plus $\Delta\theta$. This produces a value of $\Delta\lambda$ = 0.186 ± 1.053 nm. From this value and using the total number of lines observed in the interference pattern of the mode m = 2, the overall wavelength spread, $\Delta\lambda_{tot}$ was initially calculated to be $1.21\times10^{-8} \pm 6.85\times10^{-8}$ m, or equivalently 12.1 ± 68.5 nm. Using Equations (4) and (5), this resulted in $\Delta\nu = 2.8\times10^{12} \pm 2.7\times10^{13}$ Hz, $\tau_{Coh} = 3.6\times10^{-13} \pm 35.3\times10^{-13}$ s, and $l_{coh} = 1.08\times10^{-4} \pm 10.59\times10^{-4}$ m. Using λ and $\Delta\lambda$, the length of the optical cavity in the diode laser can also be calculated to be 3.50 ± 0.03 mm.

A significant factor to note is that in the final calculation of $\Delta\lambda$, $\Delta\nu$, l_{Coh}, and τ_{Coh} the final uncertainty is larger than the experimental value. This seems to be the result of propagating the uncertainty in $\Delta\lambda$ using the derivative method. However, when the fractional uncertainty is used to propagate the uncertainty in $\Delta\lambda$, the resulting value that retains rather than distorts the fractional uncertainty in the error calculations. Calculating the uncertainty in $\Delta\lambda$ by this method also changes the uncertainties in $\Delta\lambda$ l_{Coh} and τ_{Coh}. The resulting values are then $\Delta\lambda = 12.1 \pm 0.4$ nm, $\Delta\nu = 12.8 \times 10^12 \pm 3.1 \times 10^9$ Hz, l_{Coh} $= 1.08 \times 10^-4 \pm 1.2 \times 10^-7$ m, and $\tau_{Coh} = 3.6 \times 10^-13 \pm 4.0 \times 10^-16$ s.

Table II. This table shows the values of the trials taken to obtain Δx. From these values, the average and standard error were taken to obtain $\Delta x_{avg} = 0.131 \pm 0.005$ cm, which is then used to find $\Delta\lambda = 0.186 \pm 1.053$ nm.

Trial	N Lines	Total Length (cm)	Δx (cm)
1	30 ± 2	3.65 ± 0.05	0.126
2	32 ± 2	4.20 ± 0.05	0.135

V. CONCLUSION

The purpose of this experiment was to measure the wavelength spacing of a diode laser and from this measurement calculate the coherence length and coherence time of the diode laser. The experimental values for these parameters were $\Delta\lambda = 1.86 \times 10^-10 \pm 1.13 \times 10^-14$ m with a total wavelength spread of $\Delta\lambda_{tot} = 12.1 \pm 0.4$ nm resulting in a coherence length of $\tau_{Coh} = 3.6{\times}10^{-13} \pm 4{\times}10^{-16}$ s and a coherence length of $l_{Coh} = 1.08{\times}10^{-4} \pm 1.2{\times}10^{-7}$ m. While the magnitude of $\Delta\lambda$ matched the expected magnitude, the resultant τ_{Coh} and l_{Coh} deviate from the expected values by multiple orders of magnitude compared to expected values on the nm scale for l_{Coh}. These values are all based on an experimentally determined value of $\lambda = 660 \pm 6$ nm.

This large deviation beyond the experimental uncertainty points to a large imprecision in the measurement methods used at some point in the process. Because the magnitude of $\Delta\lambda$ matches the expected magnitude, this error most likely entered the process due to an underestimation of the total number of lines in the interference pattern since the single slit aperture and reflection from the spherical mirrors along with a large amount of extraneous light significantly dimmed the interference pattern and made it difficult to view the full spectrum and to count the lines near the end of the spectrum.

These results are important because they demonstrate that diode lasers, just like HeNe lasers, have their own associated coherence length and time proportional to the length of the laser's optical cavity. As expected, just as the cavity length of the diode laser is much smaller than that of a HeNe laser of comparable wavelength, the resultant coherence time and length are both much shorter.

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